Exam Quantum Physics 2

Thursday, November 8, 2007, 9:00-12:00.

Before you start, read the following:

- There are 4 problems with a total of 50 points.
- Write your name and student number on every sheet of paper.
- Write the solution of each problem on a separate sheet of paper.
- Illegible writing will be graded as incorrect.
- *Good luck!*

Problem 1 *(45 minutes; 15 points in total)*

Answer the following questions, brief and to the point:

2 pnts (a) Prove that $[J^2, S_z] = 2i\hbar(\vec{S} \times \vec{L})_z$ where $\vec{J} = \vec{L} + \vec{S}$.

2 pnts (b) Give the possible wave functions of two free electrons, taking into account the Pauli principle.

2 pnts (c) Write down the Hamiltonian of the helium atom. What is the ground-state energy, in formula and in eV, when the interaction between the electrons is neglected?

2 pnts (d) Formulate the spin-statistics theorem. Give two examples of a boson, and three examples of a fermion.

2 pnts (e) Which of the two isotopes of rubidium ($Z = 37$), ^{86}Rb or ^{87}Rb, can be used for Bose-Einstein condensation? Why?

2 pnts (f) A carbon atom has two p electrons in the outer shell. Which of the possible terms $^{2S+1}L_J$ are allowed by the exclusion principle?

2 pnts (g) Consider (time-independent, nondegenerate) perturbation theory for a Hamiltonian of the form $H = H_0 + \lambda H'$. Give the formula for the first-order correction to the energy E_0, and explain in words what it says.

1 pnt (h) Where is Schrödinger's cat?

Problem 2 (*45 minutes; 15 points in total*)

The Balmer series in hydrogen is the series of spectral lines that correspond to transitions $n' \to n = 2$.

4 pnts (a) Calculate the energy in eV, and the wavelength in nm, of the Balmer-α line ($n' = 3 \to n = 2$) and of the limit of the series $n' \to \infty$. Use $\alpha = 1/137$ and $\hbar c = 200$ eV·nm . In which part of the electromagnetic spectrum do these lines lie?

2 pnts (b) Calculate the relative difference of the wavelengths of the Balmer-α line for deuterium and for hydrogen.

Consider next the fine-structure of the hydrogen spectrum. The energies are given by

$$E_{n\ell j} = -|E_n| \left[1 + \left(\frac{Z\alpha}{n}\right)^2 \left(\frac{n}{j+\frac{1}{2}} - \frac{3}{4}\right)\right],$$

where E_n are the Bohr energies, $Z = 1$, and $j = \ell \pm 1/2$.

3 pnts (c) Discuss (no derivations!) which two physical effects are responsible for the fine-structure.

3 pnts (d) Calculate the fine-splitting of the $n = 2$ and $n = 3$ Bohr levels by giving the shifts with respect to the corresponding Bohr energies, in units of 10^{-5} Rydberg.

3 pnts (e) Give the dipole selection rules for fine-structure levels (no derivation!). Out of how many, and which, lines does the Balmer-α line consist? Make a schematic drawing of the levels involved and indicate the transitions.

Problem 3 (*35 minutes; 10 points in total*)

An electron, with mass m, is confined in a 3D cubic box with sides of length L, *i.e.* the potential is:

$$\begin{aligned} V(x,y,z) &= 0 \qquad 0 < x,y,z < L \,, \\ &= \infty \qquad x,y,z < 0 \text{ or } x,y,z > L \,. \end{aligned}$$

3 pnts (a) Give the (time-independent) Schrödinger equation. Show that the solution that obeys the proper boundary conditions is

$$\psi(x,y,z) = A\sin(k_x x)\sin(k_y y)\sin(k_z z) \,.$$

What are the conditions on k_x, k_y, and k_z? Give the corresponding energy eigenvalues E. Calculate the normalization constant A (assume that it is real and positive).

2 pnts (b) Discuss the degeneracy of the energy levels.

3 pnts (c) Now put 24 electrons in the box. Assume that they do not interact with each other. What is the lowest possible energy, in units of $\hbar^2\pi^2/(2mL^2)$?

2 pnts (d) Answer question (c) for *spinless* particles with mass m.

Problem 4 (*35 minutes; 10 points in total*)

5 pnts (a) Write during 15 minutes about the question: *What is spin?*

5 pnts (b) Write during 15 minutes about the question: *What is the difference between classical and quantum physics?*